A NOTE ON THE RELATIVE LOSS OF INFORMATION IN CONFOUNDED DESIGNS

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1. INTRODUCTION

Kshirsagar (1957) has presented a very elegant proof of the well-known property of the confounded designs that the total relative loss of information in a confounded design is always one less than the average number of blocks per replication. He has, however, restricted his results to the case of equi-replicated designs. The purpose of this note is to generalise the results to a general incomplete block binary design specified by the parameters $v, b, k_1, k_2 \ldots\ldots, k_b$, $r_1, r_2 \ldots\ldots r_v$.

2. RESULTS

Following Kshirsagar (1957), let $N=(n_{ij})$ be the incidence matrix, where $n_{ij}=1$, if the *i-th* treatment occurs in the *j-th* block, otherwise $n_{ij}=0$. Let Q_i be the adjusted yield and t_i be the effect of the *i-th* treatment. Then.

$$E(Q) = ct \tag{1}$$

where Q and t denote the column vector $(Q_1, \ldots\ldots, Q_v)$

and $(t_1, t_2 \ldots\ldots, t_v)$

respectively and

$$C = Dig(r_1, \ldots\ldots, r_v) - N Dig(1/k_1, 1/k_2, \ldots\ldots, 1/k_b) N'.$$

Let $\lambda_1, \lambda_2 \ldots\ldots \lambda_s$ be the s non-zero characteristic roots of matrix C. If l_u is column vactor and $l_u' t$ $(u=1, 2, \ldots\ldots, s)$ is estimable, then $l_u = C m_u$, where m_u is a column vector. In a randomised block design with Σk_i experimental units equally shared by the v treatments, the variance of the best estimate of $l_u' t$ is

$$\frac{1}{\bar{r}} \sigma^2 l'_u l_u \quad \ldots(2)$$

where

$$\bar{r} = \sum_{j=1}^{b} k_j / v.$$

The corresponding variance of the estimate of $l_u' t$ in the confounded design with reduced normal equation (1) is $m_u' C m_u \sigma^2$. Hence the relative information on $l_u' t$ is given by

$$\text{R.I.} = l_u' l_u / \bar{r} m_u' C m_u \qquad \ldots(3)$$

Kshirsagar (1957) has futher shown that by suitably choosing m_u, it can be easily shown that

$$\sum_{u=1}^{s} l_u' l_u / m'_u c m_u = \sum_{i=1}^{s} \lambda_i$$

Hence

$$\text{Total R.I.} = 1/\bar{r} \sum_{i=1}^{s} \lambda_i$$

Where $s < v-1$ is the number of estimable contrasts. But in a design

$$\sum_{i=1}^{s} \lambda_i = \text{trace } C = \sum_{i=1}^{v} r_i - \sum_{i=1}^{v} \sum_{j=1}^{b} n_{ij}^2 / k_j$$

$$= \sum_{j=1}^{b} k_j - b \qquad \ldots(4)$$

$$= v\bar{r} - b$$

Hence the total relative information on s contrasts is given by

$$v - b/\bar{r} \qquad \ldots(5)$$

which gives the total loss of information as

$$(v-1) - \left(v - \frac{b}{\bar{r}} \right)$$

$$= (b/\bar{r}) - 1$$

$$= \frac{vb}{\sum_{j=1}^{b} k_j} - 1. \qquad \ldots(6)$$

3. Example

In a qualitative-cum-quantitative experiment, the number of replications for the dummy treatments is generally more than those of other treatments combinations. For example, consider the experiment which has all possible combinations of 3 levels (0, 1, 2) and 3 qualities (0, 1, 2) of nitrogen and 2 levels of P_2O_5 (0, 1). The design partially confounding Q and NQ is given in Table 1.

TABLE 1

$3 \times 3 \times 2$ *qualitative-cum-quantitative experiments*

Replication I									*Replication II*								
Block I			*Block II*			*Block III*			*Block I*			*Block II*			*Block III*		
n	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*	*n*	*q*	*p*
0	—	0	0	—	0	0	—	0	0	—	0	0	—	0	0	—	0
0	—	1	0	—	1	1	—	1	0	—	1	0	—	1	0	—	1
1	2	0	1	0	0	1	1	0	1	1	0	1	2	0	1	0	0
1	2	1	0	0	1	1	1	1	1	1	1	1	2	1	1	0	1
2	1	0	2	2	0	2	0	0	2	2	0	2	0	0	2	1	0
2	1	1	2	2	1	2	0	1	2	2	1	2	0	1	2	1	1

On analysing the above design by fitting of constants by the method of least squares, it is found that the loss of information on each of 2 *df.* of Q is 1/6 and that on each of 2 *df.* of NQ is 1/2. Hence the total loss of information is 4/3. From (6) the total loss of information also works out to

$$\frac{14 \times 6}{36} - 1 = 4/3$$

where v is the number of distinct treatments.

REFERENCE

Kshirsagar, A.M. (1957) : A note on the total relative loss of information in any design. Cal. Sta. Ass. Bull., 7, 78-81.